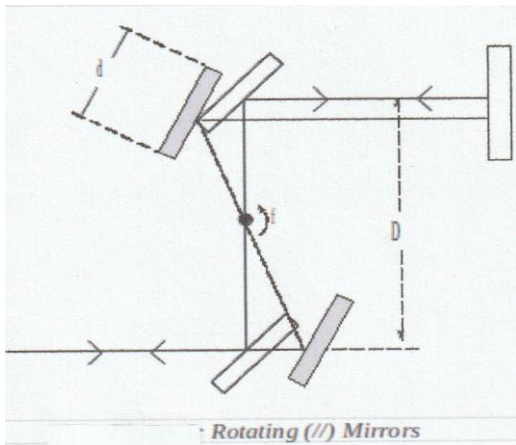


## Rapid Scan Delay by Rotating // Mirrors



In the geometry above, it can be shown that \* the pathlength of the delay beam in the Michelson set-up varies as

$$\Delta = 2\sqrt{2}D[\text{Cos}\psi\text{Sin}\theta + \text{Sin}\psi(1-\text{cos}\theta)] ,$$

where there is an angle of incidence  $\psi = 45^\circ$  near the center of the scan range, for which the fixed pathlength equals the delayed pathlength (ZPD).

Hence,

$$\Delta = 2D [\text{sin}\theta + (1-\text{Cos}\theta)] ,$$

where  $\theta = 2\pi t f$  with respect to the ZPD, and  $f =$  rotation rate

Converting to time delay  $T$  of an optical beam,

$$\mathbf{T = 2D/c[\text{sin}\theta + 1-\text{Cos}\theta]}$$

For small angles , this **exact** expression can be simplified to  **$T = 2D/c\text{Sin}\theta$**  which approximately yields

$$\mathbf{T = (4\pi Df/c)t}$$

Hence, optical delay is linear with time, for small angles.

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\* Z.A. Yasa and N.M. Amer, Optics Commun., V36, 406 (1981)

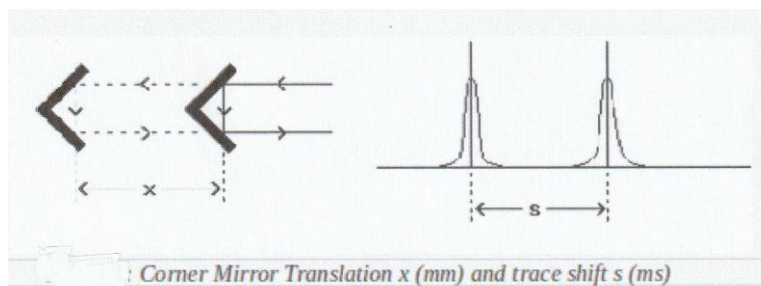
## Calibration Factor and Accuracy

Hence, the delay generated by the crystal phase-locked uniform motion of // Mirrors has a precisely defined functional form, and there is no uncertainty of mirror position (delay) during the scan. This leads to a theoretically defined **Calibration Factor (CF)**.

$$CF = T/t = 4\pi fD/c$$

Note that the CF is the same for all pulsewidth ranges (few fsecs → 100s of psecs).

In the **FR-103**, a **micrometer** on the translation stage in the fixed arm of the Michelson Interferometer set-up allows the user to shift the autocorrelation trace, thereby **verifying** the CF.



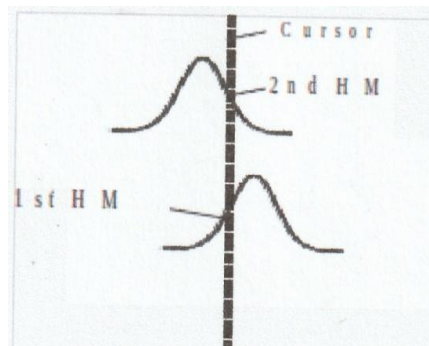
$$T/t = \frac{2x}{0.3s} (\text{ps/ms})$$

The result of such a measurement yields the theoretical value  $4\pi fD/c$ .

This unique feature of the **FR-103** facilitates unambiguous and precise pulsewidth measurements. Without such a capability for the user to check and verify the calibration, pulsewidth measurement would have uncertainty and prone to be inaccurate.

For any position of the pulse peak, the exact FWHM autocorrelation width can be experimentally determined directly as follows, without a reference to the calibration factor:

1. Set the cursor at the 2nd HM(Half Max)
2. Move the micrometer by  $s$ (mm) such that the 1st HM coincides with the cursor.
3. (FWHM)  $\Delta T = 6667 \cdot s$  (fs)  
E.g: If  $s = 0.100\text{mm}$  ( $100\mu\text{m}$ )  
→  $\Delta T = 666.7$  fs



FWHM Autocorrelation Width Measurement

## Delay Nonlinearity

Taylor series expanding the exact expression,

$$T = 2D/c[\theta - \theta^3/6 + \dots - \theta^2/2 + \theta^4/24 \dots]$$

The linear term dominates and the even terms tend to cancel out (see next page) to the first order for FWHM pulsewidth measurement .

The error term  $\theta^2/6$  decreases quadratically with  $\theta$  (shorter pulses), and leads to an error in terms of FWHM autocorrelation width ( $T_{fwhm}$ ),

$$\epsilon_s = -[cT_{fwhm}/D]^2/96$$

As shown below, the even terms (due to  $(1-\cos Q)$ ) yield an error in FWHM pulsewidth given by

$$\epsilon_c = +[cT_{fwhm}/D]^2/32$$

Hence, total nonlinearity error is :

$$\text{Delay NL Error} = [cT_{fwhm}/D]^2/48$$

Example:

For the FR-103XL,  $D=75\text{mm}$ , and for a  $T_{fwhm}=150\text{ps}$  (100ps pulse)

$$\text{Delay NL Error} = 0.75\%$$

and it can be neglected for shorter pulsewidths.

## Addendum:

### Delay NL Error due to (1-Cos $\theta$ ) term:

$$T = (2D/c)\theta(1-\theta/2)$$

$$\implies (c/D)T = 2\theta - \theta^2$$

Therefore

$$(c/D)T_{\text{hm}} = 2\theta_+ - \theta_+^2$$

$$(c/D)T_{\text{hm}} = 2\theta_- + \theta_-^2 \quad [T_{\text{hm}} = \text{Delay at half max pulsewidth}]$$

$$\implies \theta_- = -1 + \sqrt{1 + (c/D)T_{\text{hm}}}$$

$$\theta_+ = 1 - \sqrt{1 - (c/D)T_{\text{hm}}}$$

$$T_{\text{meas}} = 2D/c[\theta_+ + \theta_-]$$

$$= 2D/c\{\sqrt{1 + (c/D)T_{\text{hm}}} - \sqrt{1 - (c/D)T_{\text{hm}}}\}$$

Using

$$(1+x)^{1/2} = 1 + x/2 - x^2/8 + x^3/16 + \dots$$

$$(1-x)^{1/2} = 1 - x/2 - x^2/8 - x^3/16 + \dots$$

$\implies$

$$T_{\text{meas}} = 2T_{\text{hm}}\{1 + [(c/D)T_{\text{hm}}]^2/8\}$$

Hence

$$T_{\text{meas}} = T_{\text{fwhm}}\{1 + [(c/D)T_{\text{fwhm}}]^2/32\}$$